WAVELET-BASED CAD MODELING OF MICROSTRIP DISCONTINUITIES USING LEAST SQUARE PRONY'S METHOD

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ABSTRACT

This paper presents a sparse-based moment method (MoM) approach for the full-wave modeling of micro-strip planar structures. Rooftop multiresolution expansions, introduced as the natural extension of the conventional rooftop basis functions, are incorporated into the integral equation formulation to achieve highly sparse linear systems. The application of the fast wavelet transform (FWT) makes it possible to easily "sparsify" the existing rooftop-based CAD programs. The scattering parameters of the network under study are determined using least square Prony's method, which extracts the characteristics of the dominant-mode incident and reflected traveling waves on the port feed lines. Numerical results are presented for various examples of microstrip discontinuities.

I. INTRODUCTION

The growing sophistication of microwave circuits has substantiated a persistent need for the development of fast and efficient numerical modeling tools. In the last two decades, the method of moments (MoM) has emerged as a powerful analysis tool for the rigorous numerical modeling of microwave planar circuits and antennas. The MoM-based codes have traditionally suffered from two major shortcomings. On one hand, the practical scope of the conventional method of moments has been seriously limited due of the fullness of moment matrices. On the other hand, the poor condition numbers of these matrices usually impose a very high degree of precision required by the numerical integration of the Green's functions. Oftentimes, this amounts to a very tedious and time-consuming matrix fill process. In this paper we show how to resolve both of these two issues by incorporating multiresolution concepts into the numerical solution of integral equations. The overall result is a very fast and efficient numerical technique with significant improvement of computation time in both the matrix fill process and linear system inversion. High degrees of matrix sparseness and improved condition numbers are the two major contributing factors in this regard. More remarkable is that the fast wavelet transform enables us to easily implement the multiresolution algorithms into the existing rooftop-based MoM codes. In other words, the wavelet-based approach can be regarded as a MoM matrix "sparsifier" or code "accelerator".

Another issue addressed in this paper is the efficient calculation of the scattering parameters based on the results of numerical simulation. Different approaches have been proposed for this post-processing stage of the numerical modeling [1]-[6]. Whereas some of these methods require a full discretization of the feed lines, others use the assumption of dominant-mode traveling waves propagating on the lines. This paper presents a scheme which combines the advantages of all the aforementioned methods. The port feed lines are excited by simple ideal voltage gap generators placed far enough from the discontinuities. The feed lines are discretized as part of the overall multiresolution grid. Then, the dominant-mode traveling waves are extracted from the higher-order modes by applying least square Prony's method to the equally spaced samples of the currents on the feed lines. The scattering parameters are computed using their formal definition in terms of normalized incident and reflected wave components.

II. THEORY

This section outlines the incorporation of multi-resolution expansions into the method of moments. Fig. 1 shows the geometry of a general N-port microstrip network. The entire surface of the microstrip structure including all the feed lines are discretized initially using a high resolution uniform grid. This grid is later transformed into a multiresolution grid using the fast wavelet transform.

Standard electric field integral equations (EFIE) are derived for the unknown planar currents on various segments of the structure. These equations are solved numerically using the method of moments in conjunction with Galerkin's testing. The unknown planar currents are expanded in a rooftop multi-

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resolution basis. The two-dimensional rooftop multiresolution analysis (MRA) is constructed from the Cartesian product of a one-dimensional linear B-spline intervallic MRA in one variable and a one-dimensional Haar MRA in the other variable [7]. The 2-D scaling functions of the rooftop MRA are simply the conventional triangular rooftop basis functions, which have long been used for the moment method analysis of electromagnetic problems. Fig. 2 shows some typical rooftop multiresolution basis functions. It is important to note that unlike some older wavelet formulations [8], the rooftop MRA is inherently an intervallic expansion which satisfies the electromagnetic boundary conditions in an exact manner.

Due to the diversity of intervallic multiresolution basis functions, the numerical evaluation of the moment integrals may become a tedious process. However, using the fast wavelet transform (FWT), it is possible to confine this time-consuming task to only scaling functions at the highest resolution level of the problem. In this case, we will have a set of Toeplitz-type moment interactions among high-resolution scaling functions. This is equivalent to a conventional high resolution MoM expansion. Then, the FWT algorithm is employed to transform this scaling-only high-resolution grid into a multiresolution grid comprised of overlapping coarser scaling/wavelet sub-grids. Because of the cancellation effect characteristic to the wavelet basis functions, a wavelet-dominated moment matrix is highly sparse [7]. Such matrices are stored using one-dimensional sparse storage schemes. The resulting sparse linear system can be solved very efficiently using sparse-based iterative solvers such as the pre-conditioned biconjugate gradient (BiCG) method. This feature of multiresolution expansions makes it possible to apply the method of moments to large-scale numerical problems.

In addition to matrix sparseness and efficiency of sparse-based numerical tools, the improvement of the system condition still contributes to the computational speed in more other ways. One of these instances is the fast convergence of the BiCG solver at very few iteration cycles with very forgiving tolerance levels. In the case of microstrip-based structures, linear systems having as many as 5000 unknowns usually achieve nice convergence after less than 50 BiCG iterations. As another interesting consequence of this condition improvement, we may relax the precision required in the numerical integration of the Green's function. After all, a majority of the small magnitude matrix elements are eventually discarded after the thresholding process. The negligible elements correspond to off-diagonal moment interactions, which usually involve very difficult integration of highly oscillatory functions. Using the convolutional nature of the Green's function, we can pre-sort these redundant interactions in advance based on the relative distance of the testing and expansion basis functions, and omit them from the numerical integration process. This pre-sorting results in an enormous amount of saving in the computation time of the matrix fill process.

After the numerical solution of the problem and finding the overall current distribution, Prony's method is used to calculate the scattering parameters of the structure. Prony's method is a classical method for the exponential approximation of arbitrary functions [9]. In general, given a function f(x) expanded in the form of $f(x) \approx C_1 e^{a_1 x} + C_2 e^{a_2 x} + \ldots + C_N e^{a_N x}$, Prony's method estimates the exponents $a_1, a_2, \ldots, a_N$, as well as the amplitudes $C_1, C_2, \ldots, C_N$, when M equally spaced samples of the function are available. Due to likely numerical errors, it is preferable to have a large number of samples, i.e., $M>N$. In this case, the system will be overdetermined and must be solved in a least square sense using its singular value decomposition (SVD).

The scattering parameters of microstrip circuits and all other circuit-related quantities such as input impedances and reflection coefficients are normally defined on idealized network ports. In other words, it is assumed that only the dominant modes propagate on the feed lines, which would be the case if the voltage gap generators are located at sufficient distances from the discontinuities. The numerically computed current distribution on the feed lines, however, contains information about higher-order modes because of the full-wave modeling of the structure. This can be written in the following form:

$$I(x) = I_0^{\pm} e^{\mp j\beta_0 x} + \sum_{n=1}^{N_{modes}} I_n^{\pm} e^{\mp j\beta_n x},$$

where the 0 subscript denotes the dominant mode. Using Prony's method, we can determine the propagation constant β_0 and the dominant incident and reflected wave amplitudes $I_0^{\pm}$ at all ports. The scattering parameters are then easily calculated from their formal definition.

III. NUMERICAL RESULTS

The wavelet-based moment method formulation using the rooftop MRA and least square Prony's method as described in the previous sections has been applied to a variety of microstrip structures including various one-port, two-port and three-port microstrip discontinuities and microstrip-fed patch antennas. Due to the space limitation, in this summary we give a few examples of the numerical results concerning simple microstrip discontinuities. More comprehensive results especially dealing with more complex configurations will be presented in the symposium.

Fig. 3 shows the S_{11} scattering parameter of a right-angled microstrip bend printed on a dielectric substrate of thickness d and ε_r=2.2 with a width-to-height ratio of w/d=3.04. The wavelet-based results have been compared with a conventional MoM formulation [1]. Fig. 4 shows the S_{12} and S_{13} scattering parameters of a symmetric microstrip T junction printed on the same dielectric substrate but with w/d=1. The figure also includes the results based on a conventional MoM formulation [1]. In all cases, the voltage gap generators are placed about two guide wavelengths or more away from the feed junctions. The current distribution is sampled at least half guide wavelength away from the feed junctions. The highest resolution scaling grid is generated based on twenty cells or more per guide wavelength. This grid is transformed into a one- or two-level multiresolution grid depending on the size of the current domain. The size of the linear systems vary from about 200 to as large as several thousands for complex planar circuits.

In the examples presented, the size of the numerical problem is quite limited, and matrix sparseness and fast BiCG convergence may not appear very tempting. More complex examples will be presented, whose numerical solution will not be practically feasible without taking full advantage of such features. The major advantage we would like to emphasize in this summary is the notable reduction of matrix fill time using a wavelet-based approach as pointed out earlier. Even with very low threshold levels of 10^{-5}, it is usually possible to neglect the moment interactions among the highest resolution basis functions which are more than one wavelength apart. However, to render a more rigorous numerical procedure, this pre-sorting is carried out in an automatic manner to ensure that no anomalous interactions are excluded erroneously.

IV. CONCLUSIONS

A very efficient and fast numerical technique has been developed for the full-wave modeling of planar microstrip circuits. While preserving the accuracy and robustness of the method of moments, this wavelet-based approach results in highly sparse linear systems amenable to a host of very efficient sparse-based numerical tools. The consequent improvement of system condition numbers leads to a very fast iterative inversion process as well as a significantly speeded matrix fill process. A very important feature of this approach is that it can easily be applied to the existing MoM-based CAD programs, which utilize rooftop basis functions for the discretization of the geometry.

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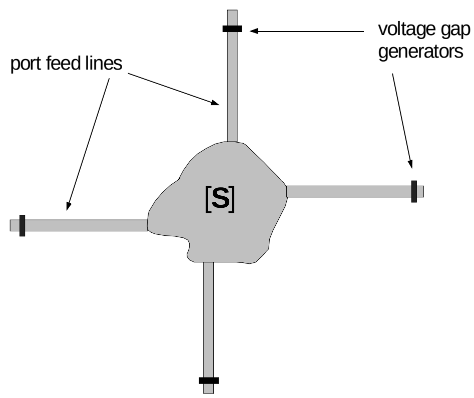

Fig. 1. Geometry of an N-port microstrip network.

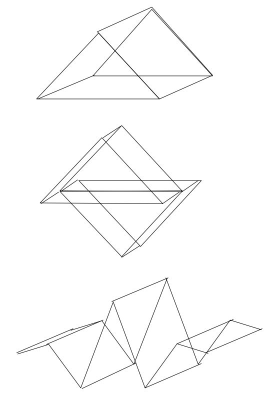

Fig. 2. Typical rooftop multiresolution basis functions.

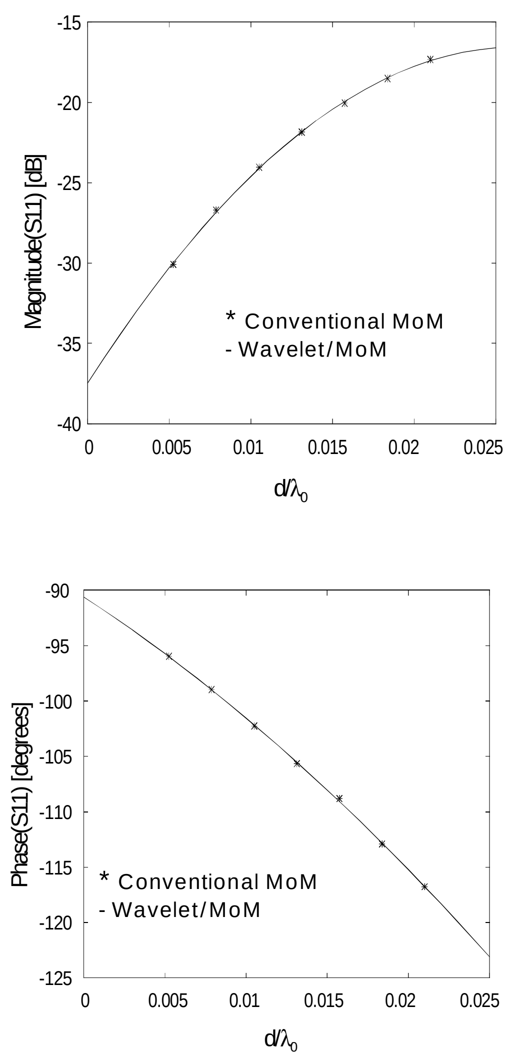

Fig. 3. S parameters of a right-angle bend.

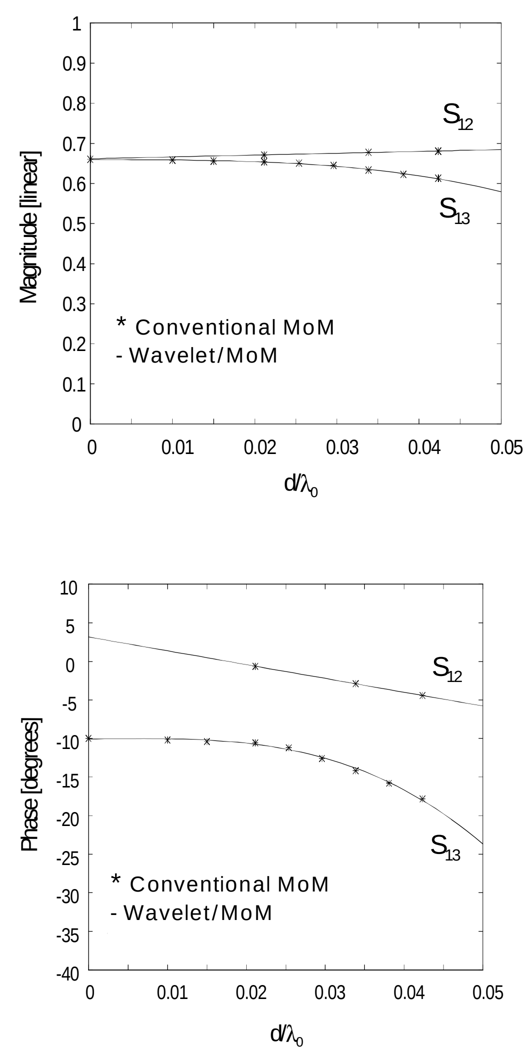

Fig. 4. S parameters of a symmetric T junction.